OCR
RECOGNISING ACHIEVEMENT

Thursday 13 June 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4757/01 Further Applications of Advanced Mathematics (FP3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4757/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

* 4 7 1 5 7 5 0 6 1 3 *

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

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- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **20** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Option 1: Vectors

1 Three points have coordinates $A(3,\ 2,\ 10)$, $B(11,\ 0,\ -3)$, $C(5,\ 18,\ 0)$, and L is the straight line through A with equation

$$\frac{x-3}{-1}=\frac{y-2}{4}=\frac{z-10}{1}.$$

(i) Find the shortest distance between the lines L and BC. **[5]**

(ii) Find the shortest distance from A to the line BC. **[6]**

A straight line passes through B and the point $P(5,\ 18,\ k)$, and intersects the line L.

(iii) Find k, and the point of intersection of the lines BP and L. **[7]**

The point D is on the line L, and AD has length 12.

(iv) Find the volume of the tetrahedron ABCD. **[6]**

Option 2: Multi-variable calculus

2 A surface has equation $z=2(x^3+y^3)+3(x^2+y^2)+12xy$.

(i) For a point on the surface at which $\dfrac{\partial z}{\partial x}=\dfrac{\partial z}{\partial y}$, show that either $y=x$ or $y=1-x$. **[5]**

(ii) Show that there are exactly two stationary points on the surface, and find their coordinates. **[7]**

(iii) The point $P(\frac{1}{2},\ \frac{1}{2},\ 5)$ is on the surface, and $Q(\frac{1}{2}+h,\ \frac{1}{2}+h,\ 5+w)$ is a point on the surface close to P. Find an approximate expression for h in terms of w. **[5]**

(iv) Find the four points on the surface at which the normal line is parallel to the vector $24\mathbf{i}+24\mathbf{j}-\mathbf{k}$. **[7]**

Option 3: Differential geometry

3 **(a)** Find the length of the arc of the polar curve $r=a(1+\cos\theta)$ for which $0\leqslant\theta\leqslant\frac{1}{2}\pi$. **[6]**

(b) A curve C has cartesian equation $y=\dfrac{x^3}{6}+\dfrac{1}{2x}$.

(i) The arc of C for which $1\leqslant x\leqslant 2$ is rotated through 2π radians about the x-axis to form a surface of revolution. Find the area of this surface. **[8]**

For the point on C at which $x=2$,

(ii) show that the radius of curvature is $\dfrac{289}{64}$, **[5]**

(iii) find the coordinates of the centre of curvature. **[5]**

Option 4: Groups

4 **(a)** The composition table for a group G of order 8 is given below.

	a	b	c	d	e	f	g	h
a	c	e	b	f	a	h	d	g
b	e	c	a	g	b	d	h	f
c	b	a	e	h	c	g	f	d
d	f	g	h	a	d	c	e	b
e	a	b	c	d	e	f	g	h
f	h	d	g	c	f	b	a	e
g	d	h	f	e	g	a	b	c
h	g	f	d	b	h	e	c	a

(i) State which is the identity element, and give the inverse of each element of G. **[3]**

(ii) Show that G is cyclic. **[4]**

(iii) Specify an isomorphism between G and the group H consisting of $\{0, 2, 4, 6, 8, 10, 12, 14\}$ under addition modulo 16. **[3]**

(iv) Show that G is not isomorphic to the group of symmetries of a square. **[2]**

(b) The set S consists of the functions $\mathrm{f}_n(x) = \dfrac{x}{1 + nx}$, for all integers n, and the binary operation is composition of functions.

(i) Show that $\mathrm{f}_m \mathrm{f}_n = \mathrm{f}_{m+n}$. **[2]**

(ii) Hence show that the binary operation is associative. **[2]**

(iii) Prove that S is a group. **[6]**

(iv) Describe one subgroup of S which contains more than one element, but which is not the whole of S. **[2]**

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

5 In this question, give probabilities correct to 4 decimal places.

A contestant in a game-show starts with one, two or three 'lives', and then performs a series of tasks. After each task, the number of lives either decreases by one, or remains the same, or increases by one. The game ends when the number of lives becomes either four or zero. If the number of lives is four, the contestant wins a prize; if the number of lives is zero, the contestant loses and leaves with nothing.

At the start, the number of lives is decided at random, so that the contestant is equally likely to start with one, two or three lives. The tasks do not involve any skill, and after every task:

- the probability that the number of lives decreases by one is 0.5,
- the probability that the number of lives remains the same is 0.05,
- the probability that the number of lives increases by one is 0.45.

This is modelled as a Markov chain with five states corresponding to the possible numbers of lives. The states corresponding to zero lives and four lives are absorbing states.

(i) Write down the transition matrix **P**. **[3]**

(ii) Show that, after 8 tasks, the probability that the contestant has three lives is 0.0207, correct to 4 decimal places. **[2]**

(iii) Find the probability that, after 10 tasks, the game has not yet ended. **[3]**

(iv) Find the probability that the game ends after exactly 10 tasks. **[3]**

(v) Find the smallest value of N for which the probability that the game has not yet ended after N tasks is less than 0.01. **[3]**

(vi) Find the limit of $\mathbf{P}^n$ as n tends to infinity. **[2]**

(vii) Find the probability that the contestant wins a prize. **[3]**

The beginning of the game is now changed, so that the probabilities of starting with one, two or three lives can be adjusted.

(viii) State the maximum possible probability that the contestant wins a prize, and how this can be achieved. **[2]**

(ix) Given that the probability of starting with one life is 0.1, and the probability of winning a prize is 0.6, find the probabilities of starting with two lives and starting with three lives. **[3]**

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Candidate forename		Candidate surname	

Centre number						Candidate number				

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1 (i)	

1 (ii)	

1 (iii)	

1 (iv)

2 (i)

2 (ii)

2 (iii)	

2 (iv)

3 (a)	

3 (b) (i)

3 (b) (ii)	

3 (b) (iii)	

4 (a) (i)	
4 (a) (ii)	
4 (a) (iii)	

4 (a) (iv)	
4 (b) (i)	
4 (b) (ii)	

4 (b) (iii)	
4 (b) (iv)	

5 (i)	
5 (ii)	
5 (iii)	

5 (iv)	
5 (v)	

5 (vi)	
5 (vii)	
5 (viii)	

5 (ix)	

GCE

Mathematics (MEI)

Advanced GCE

Unit **4757:** Further Applications of Advanced Mathematics

Mark Scheme for June 2013

Oxford Cambridge and RSA Examinations

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (eg 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g. Rules for replaced work
If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1	(i)		$\begin{pmatrix}-1\\4\\1\end{pmatrix}\times\overrightarrow{BC}=\begin{pmatrix}-1\\4\\1\end{pmatrix}\times\begin{pmatrix}-6\\18\\3\end{pmatrix}=\begin{pmatrix}-6\\-3\\6\end{pmatrix}\quad [=3\begin{pmatrix}-2\\-1\\2\end{pmatrix}]$	M1* A1	Vector product of directions	*Intention sufficient*
			Shortest distance is $\dfrac{\overrightarrow{AB}\,.\,\mathbf{d}}{\lvert\mathbf{d}\rvert}=\dfrac{\begin{pmatrix}8\\-2\\-13\end{pmatrix}.\begin{pmatrix}-2\\-1\\2\end{pmatrix}}{\sqrt{2^2+1^2+2^2}}$	M1* M1	Appropriate scalar product Evaluation of $\lvert\mathbf{d}\rvert$	*Dep* * *Dep* **
			Shortest distance is $\dfrac{40}{3}$	A1 **[5]**		
		OR	$\left[\begin{pmatrix}11-6\lambda\\18\lambda\\-3+3\lambda\end{pmatrix}-\begin{pmatrix}3-\mu\\2+4\mu\\10+\mu\end{pmatrix}\right].\begin{pmatrix}-1\\4\\1\end{pmatrix}=0$ and $\begin{pmatrix}8-6\lambda+\mu\\-2+18\lambda-4\mu\\-13+3\lambda-\mu\end{pmatrix}.\begin{pmatrix}-6\\18\\3\end{pmatrix}=0$		M1* Two appropriate scalar products	
			$81\lambda-18\mu=29,\ \ 123\lambda-27\mu=41$		A1 Two correct equations	
			$\lambda=-\dfrac{5}{3},\ \mu=-\dfrac{82}{9},\ \ \overrightarrow{XY}=\begin{pmatrix}80/9\\40/9\\-80/9\end{pmatrix}$		M1* Obtaining $\overrightarrow{XY}$	*Dep* *
			Shortest distance is $\sqrt{\left(\dfrac{80}{9}\right)^2+\left(\dfrac{40}{9}\right)^2+\left(\dfrac{80}{9}\right)^2}$		M1	*Dep* **
			Shortest distance is $\dfrac{40}{3}$		A1	

Question			Answer	Marks	Guidance	
1	(ii)		$\overrightarrow{AB}\times\overrightarrow{BC}=\begin{pmatrix}8\\-2\\-13\end{pmatrix}\times\begin{pmatrix}-6\\18\\3\end{pmatrix}=\begin{pmatrix}228\\54\\132\end{pmatrix}$	M1* A2	Appropriate vector product Give A1 if one error	
			$\left\lvert\overrightarrow{AB}\times\overrightarrow{BC}\right\rvert=\sqrt{228^2+54^2+132^2}$	M1*		*Dep* *
			$\left\lvert\overrightarrow{BC}\right\rvert=\sqrt{6^2+18^2+3^2}$			
			Shortest distance is $\dfrac{\left\lvert\overrightarrow{AB}\times\overrightarrow{BC}\right\rvert}{\left\lvert\overrightarrow{BC}\right\rvert}=\sqrt{\dfrac{72324}{369}}$	M1		*Dep* **
			Shortest distance is 14	A1 **[6]**		*Sign error in vector product can earn M1A1M1M1A1*
		OR	$\left[\begin{pmatrix}11-6\lambda\\18\lambda\\-3+3\lambda\end{pmatrix}-\begin{pmatrix}3\\2\\10\end{pmatrix}\right].\begin{pmatrix}-6\\18\\3\end{pmatrix}=0$		M1* Allow one error A1	
					M1* Obtaining a value of λ	*Dep* *
			$\lambda=\dfrac{1}{3}$		A1	
			Shortest distance is $\sqrt{(6)^2+(4)^2+(-12)^2}$		M1	*Dep* **
			Shortest distance is 14		A1	

Question			Answer	Marks	Guidance	
1	(iii)		$\begin{pmatrix}11\\0\\-3\end{pmatrix}+\lambda\begin{pmatrix}-6\\18\\k+3\end{pmatrix}=\begin{pmatrix}3\\2\\10\end{pmatrix}+\mu\begin{pmatrix}-1\\4\\1\end{pmatrix}$			
			$11-6\lambda=3-\mu$ $18\lambda=2+4\mu$	M1 A1	Allow one error Two correct equations	Must use different parameters
			$\lambda=5,\quad \mu=22$	A1		
			$-3+\lambda(k+3)=10+\mu$ $k=4$	M1 A1	Obtaining a value of *k*	*Other methods possible* *(e.g. distance between lines is 0)*
			Point of intersection is $\begin{pmatrix}3\\2\\10\end{pmatrix}+22\begin{pmatrix}-1\\4\\1\end{pmatrix}$	M1		
			Point of intersection is $(-19,\ 90,\ 32)$	A1 **[7]**		
1	(iv)		$\left\lvert\begin{pmatrix}-1\\4\\1\end{pmatrix}\right\rvert=\sqrt{18}$, so $\overrightarrow{AD}=(\pm)\dfrac{12}{\sqrt{18}}\begin{pmatrix}-1\\4\\1\end{pmatrix}=2\sqrt{2}\begin{pmatrix}-1\\4\\1\end{pmatrix}$	M1* A1	Obtaining $\overrightarrow{AD}$ or D	
			Volume is $\frac{1}{6}(\overrightarrow{AB}\times\overrightarrow{AC})\cdot\overrightarrow{AD}$	M1*	Appropriate scalar triple product	
			$=\dfrac{1}{6}\left[\begin{pmatrix}8\\-2\\-13\end{pmatrix}\times\begin{pmatrix}2\\16\\-10\end{pmatrix}\right].(2\sqrt{2})\begin{pmatrix}-1\\4\\1\end{pmatrix}$	A1 ft	Correct expression	*Can be implied*
			$=\dfrac{\sqrt{2}}{3}\begin{pmatrix}228\\54\\132\end{pmatrix}\cdot\begin{pmatrix}-1\\4\\1\end{pmatrix}=\dfrac{\sqrt{2}}{3}(120)$	M1	Evaluating scalar triple product	*Dep* **
			$=40\sqrt{2}$	A1 **[6]**	Accept 56.6	

Question			Answer	Marks	Guidance	
2	(i)		$\frac{\partial z}{\partial x}=6x^2+6x+12y$	B1		
			$\frac{\partial z}{\partial y}=6y^2+6y+12x$	B1		
			If $\frac{\partial z}{\partial x}=\frac{\partial z}{\partial y}$, $6x^2+6x+12y=6y^2+6y+12x$			
			$x^2-y^2-x+y=0$			
			$(x-y)(x+y-1)=0$	M1	Identifying factor $(x-y)$	*SC* If M0, then give
			$y=x$ or $y=1-x$	E1E1		B1 for verifying $y=x$ B1 for verifying $y=1-x$
				[5]		
2	(ii)		$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial y}=0$	M1		Can be implied
			If $y=x$ then $6x^2+6x+12x=0$	M1	Obtaining quadratic in x (or y)	Or quartic, and factorising as x(linear)(quadratic)
			$x=0,\ -3$	M1	Obtaining a non-zero value of x	
			Stationary points $(0,\ 0,\ 0)$ and $(-3,\ -3,\ 54)$	B1A1	Condone $(0,\ 0)$ for B1	
			If $y=1-x$ then $6x^2+6x+12(1-x)=0$			
			$x^2-x+2=0$	M1	Obtaining quadratic with no real roots	
			Which has no real roots ($D=-7<0$)	A1	Correctly shown	Just stating 'No real roots' M1A0
				[7]		
2	(iii)		At P, $\frac{\partial z}{\partial x}=\frac{21}{2}$, $\frac{\partial z}{\partial y}=\frac{21}{2}$	M1 A1	Substituting into $\frac{\partial z}{\partial x}$ or $\frac{\partial z}{\partial y}$	Correct value, or substitution seen
			$\delta z\approx\frac{\partial z}{\partial x}\delta x+\frac{\partial z}{\partial y}\delta y$	M1		
			$w\approx\frac{21}{2}h+\frac{21}{2}h$	A1 ft		
			$h\approx\frac{w}{21}$	A1		

Question			Answer	Marks	Guidance	
				[5]		
2	(iv)		$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} = 24$	M1	Allow sign error	24λ is M0 unless $\lambda = \pm 1$ appears later
			If $y = x$ then $6x^2 + 6x + 12x = 24$ $x = 1,\ -4$	M1	Obtaining quadratic in x (or y)	Or quartic, and one linear factor
			Points $(1,\ 1,\ 22)$ and $(-4,\ -4,\ 32)$	A1A1	If neither correct, give A1 for $x = 1,\ -4$	
			If $y = 1 - x$ then $6x^2 + 6x + 12(1-x) = 24$ $x = 2,\ -1$	M1	Obtaining quadratic in x (or y)	Or third linear factor of quartic
			Points $(2,\ -1,\ 5)$ and $(-1,\ 2,\ 5)$	A1A1 [7]	If neither correct, give A1 for $x = 2,\ -1$	
3	(a)		$r^2 + \left(\frac{\mathrm{d}r}{\mathrm{d}\theta}\right)^2 = a^2(1+\cos\theta)^2 + (-a\sin\theta)^2$	B1	Condone ... $+(a\sin\theta)^2$	
			$= a^2(1 + 2\cos\theta + \cos^2\theta + \sin^2\theta) = 2a^2(1+\cos\theta)$		or $4a^2\cos^4\frac{1}{2}\theta + 4a^2\sin^2\frac{1}{2}\theta\cos^2\frac{1}{2}\theta$	
				M1	Using $1+\cos\theta = 2\cos^2\frac{1}{2}\theta$	
			$= 4a^2\cos^2\frac{1}{2}\theta$	A1		
			Arc $\int\sqrt{r^2 + \left(\frac{\mathrm{d}r}{\mathrm{d}\theta}\right)^2}\,\mathrm{d}\theta = \int_0^{\frac{1}{2}\pi} 2a\cos\frac{1}{2}\theta\,\mathrm{d}\theta$	M1	For $\int\sqrt{r^2 + \left(\frac{\mathrm{d}r}{\mathrm{d}\theta}\right)^2}\,\mathrm{d}\theta$ in terms of θ	Limits not required
			$= \left[\ 4a\sin\frac{1}{2}\theta\ \right]_0^{\frac{1}{2}\pi}$	A1	For $4a\sin\frac{1}{2}\theta$	
			$= 2\sqrt{2}\,a$	A1 [6]		

Question			Answer	Marks	Guidance	
3	(b)	(i)	$1+\left(\frac{dy}{dx}\right)^2=1+\left(\frac{x^2}{2}-\frac{1}{2x^2}\right)^2$	B1		
			$=\frac{x^4}{4}+\frac{1}{2}+\frac{1}{4x^4}$	M1		
			$=\left(\frac{x^2}{2}+\frac{1}{2x^2}\right)^2$	A1		
			Area is $\int 2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$	M1*		
			$=\int_1^2 2\pi\left(\frac{x^3}{6}+\frac{1}{2x}\right)\left(\frac{x^2}{2}+\frac{1}{2x^2}\right)dx$	A1 ft	Integral expression including limits	
			$=2\pi\int_1^2\left(\frac{x^5}{12}+\frac{x}{3}+\frac{1}{4x^3}\right)dx$	M1	Obtaining integrable form	*Dep* *
			$=2\pi\left[\frac{x^6}{72}+\frac{x^2}{6}-\frac{1}{8x^2}\right]_1^2$	A1	Allow one error	
			$=\frac{47\pi}{16}$	A1 **[8]**		

Question			Answer	Marks	Guidance	
3	(b)	(ii)	$\frac{d^2y}{dx^2}=x+\frac{1}{x^3}\quad\left(=\frac{17}{8}\right)$	B1		
			$\rho=\dfrac{\left(\frac{x^2}{2}+\frac{1}{2x^2}\right)^3}{x+\frac{1}{x^3}}$	M1 A1 ft	Using formula for ρ or κ Correct expression for ρ or κ	
			$=\dfrac{\left(1+\left(\frac{15}{8}\right)^2\right)^{\frac{3}{2}}}{2+\frac{1}{8}}=\dfrac{\left(\frac{17}{8}\right)^3}{\frac{17}{8}}$	A1 ft	Correct numerical expression for ρ	
			$=\frac{289}{64}$	E1	Correctly shown	
				[5]		
3	(b)	(iii)	$\frac{dy}{dx}=\frac{15}{8}$, so unit normal is $\frac{1}{17}\begin{pmatrix}-15\\8\end{pmatrix}$	M1 A1	Obtaining a normal vector Correct unit normal	Allow M1 for $\begin{pmatrix}\pm8\\\pm15\end{pmatrix}$ or $\begin{pmatrix}\pm15\\\pm8\end{pmatrix}$
			$\mathbf{c}=\begin{pmatrix}2\\19/12\end{pmatrix}+\frac{289}{64}\begin{pmatrix}-15/17\\8/17\end{pmatrix}$	M1	Allow sign errors	Must use a unit vector
			Centre of curvature is $\left(-\frac{127}{64},\ \frac{89}{24}\right)$	A1A1		
				[5]		

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th colspan="2">Guidance</th></tr>
<tr><td>4</td><td>(a)</td><td>(i)</td><td>Identity is e
<table><tr><td>Element</td><td>a</td><td>b</td><td>c</td><td>d</td><td>e</td><td>f</td><td>g</td><td>h</td></tr><tr><td>Inverse</td><td>b</td><td>a</td><td>c</td><td>g</td><td>e</td><td>h</td><td>d</td><td>f</td></tr></table></td><td>B1
B2
[3]</td><td>Give B1 for four correct</td><td></td></tr>
<tr><td>4</td><td>(a)</td><td>(ii)</td><td>$d^2 = a,\ \ d^4 = c$

Hence d has order 8, and G is cyclic</td><td>M1
A1
A1

E1
[4]</td><td>Finding powers of an element
Identifying d (or f or g or h) as a generator
Or $f^2 = b,\ \ f^4 = c$
Or $g^2 = b,\ \ g^4 = c$
Or $h^2 = a,\ \ h^4 = c$
Correctly shown</td><td>At least fourth power
Implies previous M1</td></tr>
<tr><td>4</td><td>(a)</td><td>(iii)</td><td><table><tr><td>H</td><td>0</td><td>2</td><td>4</td><td>6</td><td>8</td><td>10</td><td>12</td><td>14</td></tr><tr><td>G</td><td>e</td><td>d</td><td>a</td><td>f</td><td>c</td><td>h</td><td>b</td><td>g</td></tr><tr><td>or</td><td>e</td><td>f</td><td>b</td><td>d</td><td>c</td><td>g</td><td>a</td><td>h</td></tr><tr><td>or</td><td>e</td><td>g</td><td>b</td><td>h</td><td>c</td><td>f</td><td>a</td><td>d</td></tr><tr><td>or</td><td>e</td><td>h</td><td>a</td><td>g</td><td>c</td><td>d</td><td>b</td><td>f</td></tr></table></td><td>B1
B1
B1
[3]</td><td>For $e \leftrightarrow 0$ and $c \leftrightarrow 8$
For $\{d, f, g, h\} \leftrightarrow \{2, 6, 10, 14\}$
For a fully correct isomorphism</td><td>In any order</td></tr>
<tr><td>4</td><td>(a)</td><td>(iv)</td><td>Rotations have order 2 or 4
Reflections have order 2

There is no element of order 8
Hence not isomorphic</td><td>B1

E1
[2]</td><td>Correct statement about rotations and/or reflections which implies non-IM
Or More than one element of order 2
Or Not commutative
Fully correct explanation</td><td>*Or* (4) reflections (and 180° rotation) have order 2
Or composition of reflections (or 90° rotation and reflection) is not commutative
Dependent on previous B1</td></tr>
</table>

Question			Answer	Marks	Guidance	
4	(b)	(i)	$\mathrm{f}_m\mathrm{f}_n(x) = \dfrac{\dfrac{x}{1+nx}}{1+m\left(\dfrac{x}{1+nx}\right)}$	M1	Composition of functions	In either order
			$= \dfrac{x}{1+nx+mx} = \dfrac{x}{1+(m+n)x} = \mathrm{f}_{m+n}(x)$	E1	Correctly shown	E0 if in wrong order
				[2]		
4	(b)	(ii)	$(\mathrm{f}_m\mathrm{f}_n)\mathrm{f}_p = \mathrm{f}_{m+n}\mathrm{f}_p = \mathrm{f}_{m+n+p}$	M1	Combining three functions	
			$\mathrm{f}_m(\mathrm{f}_n\mathrm{f}_p) = \mathrm{f}_m\mathrm{f}_{n+p} = \mathrm{f}_{m+n+p}$			
			Hence *S* is associative	E1	Correctly shown	M1E1 bod for $(\mathrm{f}_m\mathrm{f}_n)\mathrm{f}_p = \mathrm{f}_{m+n+p} = \mathrm{f}_m(\mathrm{f}_n\mathrm{f}_p)$
				[2]		
4	(b)	(iii)	For any f_m , f_n in *S*, $\mathrm{f}_m\mathrm{f}_n = \mathrm{f}_{m+n}$	M1	Referring to this in context	
			$\mathrm{f}_m\mathrm{f}_n$ is in *S* (so *S* is closed)	A1		
			Identity is f_0	B1	B0 for *x* B1 for $n=0$	
			Inverse of f_n is f_{-n}	B1		
			since $\mathrm{f}_n\mathrm{f}_{-n} = \mathrm{f}_{n-n} = \mathrm{f}_0$	B1		
			S is also associative, and hence is a group	E1	Closure, associativity, identity and inverses must all be mentioned in (iii)	Dependent on previous 5 marks
				[6]		
4	(b)	(iv)	$\{\mathrm{f}_{2n}\}$ for all integers *n*	B2	Or $\{\mathrm{f}_{3n}\}$, etc	
					Give B1 for multiples of 2 (or 3, etc) but not completely correctly described	e.g. $\{\mathrm{f}_0 , \mathrm{f}_2 , \mathrm{f}_4 , \mathrm{f}_6 , \ldots\}$
				[2]		

Question			Answer	Marks	Guidance	
5	(i)		***Pre-multiplication by transition matrix*** $$\mathbf{P} = \begin{pmatrix} 1 & 0.5 & 0 & 0 & 0 \\ 0 & 0.05 & 0.5 & 0 & 0 \\ 0 & 0.45 & 0.05 & 0.5 & 0 \\ 0 & 0 & 0.45 & 0.05 & 0 \\ 0 & 0 & 0 & 0.45 & 1 \end{pmatrix}$$	B3 **[3]**	Allow tolerance of ± 0.0001 in probabilities throughout this question Give B2 for four columns correct Give B1 for two columns correct	
5	(ii)		$$\mathbf{P}^8 \begin{pmatrix} 0 \\ 1/3 \\ 1/3 \\ 1/3 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.5042 \\ 0.0230 \\ 0.0278 \\ \mathbf{0.02071} \\ 0.4242 \end{pmatrix}$$ P(3 lives) = 0.0207 (4 dp)	M1 E1 **[2]**	For $\mathbf{P}^8$ (allow $\mathbf{P}^7$ or $\mathbf{P}^9$) and initial column matrix Correctly shown	
5	(iii)		Let q(n) = P(not yet ended after n tasks) $$= \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \end{pmatrix} \mathbf{P}^n \begin{pmatrix} 0 \\ 1/3 \\ 1/3 \\ 1/3 \\ 0 \end{pmatrix}$$ $\text{q}(10) = 0.0371$	M1 M1 A1 **[3]**	Obtaining probabilities after 10 tasks Adding probabilities of 1, 2, 3 lives	Allow M1 for using $\mathbf{P}^9$ or $\mathbf{P}^{11}$

Question			Answer	Marks	Guidance	
5	(iv)		$q(9)-q(10)$ $=0.05072-0.03709$ $=0.0136$	M1 M1 A1 **[3]**	Using $q(9)$ and $q(10)$ Evaluating $q(9)$	
		OR	$\mathbf{P}^9\begin{pmatrix}0\\ 1/3\\ 1/3\\ 1/3\\ 0\end{pmatrix}=\begin{pmatrix}.\\ 0.01506\\ .\\ 0.01355\\ .\end{pmatrix}$ $0.01506\times0.5+0.01355\times0.45$ $=0.0136$		M1 Probs of 1 and 3 lives after 9 tasks M1 A1	
5	(v)		$q(13)=0.01374$ $q(14)=0.00998$ Smallest N is 14	M1 M1 A1 **[3]**	Evaluating $q(n)$ for some $n>10$ Consecutive values each side of 0.01 Must be clear that their answer is 14	Just $N=14$ www earns B3
5	(vi)		$\mathbf{P}^n\to\begin{pmatrix}1&0.7880&0.5525&0.2908&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0.2120&0.4475&0.7092&1\end{pmatrix}=\mathbf{L}$	B2 **[2]**	Give B1 for any element correct to 3 dp (other than 0 or 1)	
5	(vii)		$\mathbf{L}\begin{pmatrix}0\\ 1/3\\ 1/3\\ 1/3\\ 0\end{pmatrix}=\begin{pmatrix}0.5438\\0\\0\\0\\0.4562\end{pmatrix}$ P(wins a prize) = 0.4562	M1M1 A1 **[3]**	Using **L** and the initial column matrix	

Question			Answer	Marks	Guidance	
5	(viii)		Maximum probability is 0.7092 Always start with 3 lives	B1 ft B1 **[2]**		
5	(ix)		$\mathbf{L}\begin{pmatrix}0\\0.1\\p\\q\\0\end{pmatrix}=\begin{pmatrix}0.4\\0\\0\\0\\0.6\end{pmatrix}$ $0.7880\times0.1+0.5525p+0.2908(0.9-p)=0.4$ P(2 lives) = 0.2273 , P(3 lives) = 0.6727	M1 M1 A1 **[3]**	Or $0.0212+0.4475p+0.7092(0.9-p)=0.6$ Obtaining a value for p or q Accept values rounding to 0.227, 0.673	Allow use of $p+q=1$
5			***Post-multiplication by transition matrix***		Allow tolerance of ±0.0001 in probabilities throughout this question	
5	(i)		$\mathbf{P}=\begin{pmatrix}1&0&0&0&0\\0.5&0.05&0.45&0&0\\0&0.5&0.05&0.45&0\\0&0&0.5&0.05&0.45\\0&0&0&0&1\end{pmatrix}$	B3 **[3]**	Give B2 for four rows correct Give B1 for two rows correct	
5	(ii)		$\begin{pmatrix}0&\frac{1}{3}&\frac{1}{3}&\frac{1}{3}&0\end{pmatrix}\mathbf{P}^8$ $=\begin{pmatrix}0.5042&0.0230&0.0278&\mathbf{0.02071}&0.4242\end{pmatrix}$ P(3 lives) = 0.0207 (4 dp)	M1 E1 **[2]**	For $\mathbf{P}^8$ (allow $\mathbf{P}^7$ or $\mathbf{P}^9$) and initial row matrix Correctly shown	

Question			Answer	Marks	Guidance	
5	(iii)		Let q(n) = P(not yet ended after n tasks) $= \begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix} \mathbf{P}^n \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$ $q(10) = 0.0371$	M1 M1 A1 **[3]**	Obtaining probabilities after 10 tasks Adding probabilities of 1, 2, 3 lives	Allow M1 for using $\mathbf{P}^9$ or $\mathbf{P}^{11}$
5	(iv)		$q(9) - q(10)$ $= 0.05072 - 0.03709$ $= 0.0136$	M1 M1 A1 **[3]**	Using $q(9)$ and $q(10)$ Evaluating $q(9)$	
		OR	$\begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix} \mathbf{P}^9$ $= \begin{pmatrix} . & 0.01506 & . & 0.01355 & . \end{pmatrix}$ $0.01506 \times 0.5 + 0.01355 \times 0.45$ $= 0.0136$		M1 Probs of 1 and 3 lives after 9 tasks M1 A1	
5	(v)		$q(13) = 0.01374$ $q(14) = 0.00998$ Smallest N is 14	M1 M1 A1 **[3]**	Evaluating q(n) for some $n > 10$ Consecutive values each side of 0.01 Must be clear that their answer is 14	Just $N = 14$ www earns B3
5	(vi)		$\mathbf{P}^n \to \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.7880 & 0 & 0 & 0 & 0.2120 \\ 0.5525 & 0 & 0 & 0 & 0.4475 \\ 0.2908 & 0 & 0 & 0 & 0.7092 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} = \mathbf{L}$	B2 **[2]**	Give B1 for any element correct to 3 dp (other than 0 or 1)	

Question			Answer	Marks	Guidance	
5	(vii)		$\begin{pmatrix} 0 & \tfrac{1}{3} & \tfrac{1}{3} & \tfrac{1}{3} & 0 \end{pmatrix}\mathbf{L}$ $=\begin{pmatrix} 0.5438 & 0 & 0 & 0 & 0.4562 \end{pmatrix}$ P(wins a prize) = 0.4562	M1M1 A1 **[3]**	Using **L** and the initial row matrix	
5	(viii)		Maximum probability is 0.7092 Always start with 3 lives	B1 ft B1 **[2]**		
5	(ix)		$\begin{pmatrix} 0 & 0.1 & p & q & 0 \end{pmatrix}\mathbf{L}$ $=\begin{pmatrix} 0.4 & 0 & 0 & 0 & 0.6 \end{pmatrix}$ $0.7880\times 0.1+0.5525p+0.2908(0.9-p)=0.4$ P(2 lives) = 0.2273 , P(3 lives) = 0.6727	 M1 M1 A1 **[3]**	Or $0.0212+0.4475p+0.7092(0.9-p)=0.6$ Accept values rounding to 0.227, 0.673	Allow use of $p+q=1$

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
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CB1 2EU

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GCE

Mathematics (MEI)

Advanced GCE **A2 7895-8**

Advanced Subsidiary GCE **AS 3895-8**

OCR Report to Centres

June 2013

4757 Further Applications of Advanced Mathematics (FP3)

General Comments

Each of the five questions contained parts which proved to be accessible to the great majority of candidates, as well as parts which turned out to be more challenging. The candidates performed similarly in all questions, and the average mark for each question was about 17 out of 24. Candidates had to choose three questions to answer; questions 1 and 2 were the most popular, and question 3 was the least popular.

Comments on Individual Questions

1) ***Vectors***

In part (i) most candidates knew the standard formula for the shortest distance between skew lines and could apply it accurately. Throughout this question sign errors in answers were surprisingly common, both in the evaluation of vector products and in copying a vector from one line to the next.

In part (ii), the formula for the shortest distance from a point to a line was less well known. Many candidates completed the work confidently and efficiently, but common mistakes included using a scalar product instead of a vector product, as well as the usual sign errors.

In part (iii) the method for finding the point of intersection of two lines was well understood, and this was very often answered correctly.

In part (iv) most candidates gave the volume of the tetrahedron correctly as a scalar triple product. To make further progress it was necessary to find the vector **AD** of length 12. Very many candidates were unable to do this, often confusing **AD** with the position vector of D. Some just substituted the scalar 12 for the vector **AD**.

2) ***Multi-variable calculus***

In part (i) the partial differentiation was done well; then most candidates verified that $y = x$ and $y = 1 - x$ both made the two derivatives equal. This did not rule out other possibilities, and to earn full marks candidates needed to do more. The usual way to do this was to identify a factor $(y - x)$; some found an alternative method applying the quadratic formula to $y^2 - y + (x - x^2) = 0$.

In part (ii) almost every candidate knew that stationary points occurred when the partial derivatives were both zero. Most candidates then used the results of part (i) to obtain two quadratic equations and hence find the stationary points. Marks were sometimes lost through arithmetic slips, and especially for not showing convincingly that one of the quadratic equations had no real roots. Several candidates did not use the results of part (i) and obtained a quartic equation, some managing to factorise this correctly.

In part (iii), the application of partial derivatives to small changes was quite well understood. Having obtained $w \approx 21h$ many candidates lost the final mark by failing to give h in terms of w.

In part (iv), most candidates stated that the partial derivatives were both equal to 24, and proceeded in a similar way to part (ii). This was often completed successfully, but sometimes spoilt by careless slips. Those who formed a quartic equation here were rarely able to solve it.

3) ***Differential geometry***

In part (a) most candidates could write down a correct integral expression for the arc length of the polar curve. Further progress required the use of half-angle formulae; while some could do this confidently, many others did not know how to proceed.

In part (b) (i) it was essential to express $1 + (\mathrm{d}y/\mathrm{d}x)^2$ as a perfect square. Many candidates were able to do this, and often went on to obtain the correct value for the curved surface area.

In parts (b)(ii) and (b)(iii) the concepts of radius of curvature and centre of curvature were well understood, and many candidates answered both parts correctly.

4) ***Groups***

Most candidates gave the identity and inverses correctly in part (a) (i) and established a generator for *G* in part (a)(ii).

In part (a)(iii) candidates who considered powers of one generator from each group were able to obtain an isomorphism easily. Some thought it necessary to write out the composition table for *H*, and many matched up elements of the same order without further consideration.

Part (a) (iv) turned out to be the most difficult item on this question paper, and most candidates did not score any marks. A statement such as 'The group of symmetries of a square is not cyclic' must be justified by something like 'The rotations have orders 1, 2 or 4 and the reflections have order 2'. Similarly 'The group of symmetries is not abelian' must be justified by giving an example of two transformations which do not commute. Nevertheless, there were some excellent explanations. Quite a few nearly correct answers were spoilt by confusing elements of order 2 with self-inverse elements, in statements such as '*G* has only one self-inverse element' when there are in fact two (*c* and *e*).

In part (b) (i) most candidates established the result correctly, although some combined the functions in the wrong order. A few candidates simply multiplied $\mathrm{f}_\mathrm{m}(x)$ by $\mathrm{f}_n(x)$.

Most candidates could prove associativity in part (b)(ii), although many confused it with commutativity.

In part (b)(iii) most were able to complete the proof that *S* is a group.

In part (b)(iv) the (correct) subgroup most often given was the functions f_n for all even integers *n*. Most candidates did not score any marks in this part, with many thinking that $\{ \mathrm{f}_0 \;\; \mathrm{f}_1 \;\; \mathrm{f}_{-1} \}$ was a subgroup.

5) ***Markov chains***

Almost every candidate who attempted this question demonstrated competence in using their calculator to handle the matrices. In parts (i), (ii) and (iii) the great majority wrote down a correct transition matrix and used it convincingly to obtain the required probabilities. The only common error was the omission of the two 1's in the transition matrix.

Part (iv) turned out to be quite challenging; most candidates started correctly by finding probabilities after 9 tasks. However, these probabilities were often used wrongly, for example multiplying the probability that the game has not ended after 9 tasks by the probability that it has ended after 10 tasks.

Part (v) was answered well; the probability that the game has not ended after 14 tasks is only just less than 0.01, and rounding errors very often led to a wrong answer of 15.

In parts (vi) and (vii) most candidates obtained the limiting matrix and used it correctly to find the probability of winning. Some did not write down the limit of $\mathbf{P}^n$ but gave instead the limiting state probabilities.

The situation in part (viii) was well understood, with almost every candidate realising that the maximum probability of winning would occur when the contestant always starts with three lives, and most gave the corresponding probability correctly. A common wrong answer was 0.45 (which is the probability of winning after the first task).

In part (ix) most candidates started with a correct matrix equation, and a good number obtained the correct starting probabilities. A fairly common error was to use **P** instead of the limiting matrix.

Unit level raw mark and UMS grade boundaries June 2013 series

AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

GCE Mathematics (MEI)

			Max Mark	a	b	c	d	e	u
4751/01	(C1) MEI Introduction to Advanced Mathematics	Raw	72	62	56	51	46	41	0
		UMS	100	80	70	60	50	40	0
4752/01	(C2) MEI Concepts for Advanced Mathematics	Raw	72	54	48	43	38	33	0
		UMS	100	80	70	60	50	40	0
4753/01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	58	52	46	40	33	0
4753/02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753/82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4753	(C3) MEI Methods for Advanced Mathematics with Coursework	UMS	100	80	70	60	50	40	0
4754/01	(C4) MEI Applications of Advanced Mathematics	Raw	90	66	59	53	47	41	0
		UMS	100	80	70	60	50	40	0
4755/01	(FP1) MEI Further Concepts for Advanced Mathematics	Raw	72	63	57	51	45	40	0
		UMS	100	80	70	60	50	40	0
4756/01	(FP2) MEI Further Methods for Advanced Mathematics	Raw	72	61	54	48	42	36	0
		UMS	100	80	70	60	50	40	0
4757/01	(FP3) MEI Further Applications of Advanced Mathematics	Raw	72	60	52	44	36	28	0
		UMS	100	80	70	60	50	40	0
4758/01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	62	56	51	46	40	0
4758/02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758/82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4758	(DE) MEI Differential Equations with Coursework	UMS	100	80	70	60	50	40	0
4761/01	(M1) MEI Mechanics 1	Raw	72	57	49	41	33	25	0
		UMS	100	80	70	60	50	40	0
4762/01	(M2) MEI Mechanics 2	Raw	72	50	43	36	29	22	0
		UMS	100	80	70	60	50	40	0
4763/01	(M3) MEI Mechanics 3	Raw	72	64	56	48	41	34	0
		UMS	100	80	70	60	50	40	0
4764/01	(M4) MEI Mechanics 4	Raw	72	56	49	42	35	29	0
		UMS	100	80	70	60	50	40	0
4766/01	(S1) MEI Statistics 1	Raw	72	55	48	41	35	29	0
		UMS	100	80	70	60	50	40	0
4767/01	(S2) MEI Statistics 2	Raw	72	58	52	46	41	36	0
		UMS	100	80	70	60	50	40	0
4768/01	(S3) MEI Statistics 3	Raw	72	61	55	49	44	39	0
		UMS	100	80	70	60	50	40	0
4769/01	(S4) MEI Statistics 4	Raw	72	56	49	42	35	28	0
		UMS	100	80	70	60	50	40	0
4771/01	(D1) MEI Decision Mathematics 1	Raw	72	58	52	46	40	35	0
		UMS	100	80	70	60	50	40	0
4772/01	(D2) MEI Decision Mathematics 2	Raw	72	58	52	46	41	36	0
		UMS	100	80	70	60	50	40	0
4773/01	(DC) MEI Decision Mathematics Computation	Raw	72	46	40	34	29	24	0
		UMS	100	80	70	60	50	40	0
4776/01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	56	50	44	38	31	0
4776/02	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776/82	(NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
4776	(NM) MEI Numerical Methods with Coursework	UMS	100	80	70	60	50	40	0
4777/01	(NC) MEI Numerical Computation	Raw	72	55	47	39	32	25	0
		UMS	100	80	70	60	50	40	0
4798/01	(FPT) Further Pure Mathematics with Technology	Raw	72	57	49	41	33	26	0
		UMS	100	80	70	60	50	40	0

GCE Statistics (MEI)

			Max Mark	a	b	c	d	e	u
G241/01	(Z1) Statistics 1	Raw	72	55	48	41	35	29	0
		UMS	100	80	70	60	50	40	0
G242/01	(Z2) Statistics 2	Raw	72	55	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
G243/01	(Z3) Statistics 3	Raw	72	56	48	41	34	27	0
		UMS	100	80	70	60	50	40	0